

Prediction for CP Violation via Electric Dipole Moment of τ Lepton in $\gamma\gamma \rightarrow \tau^+\tau^-$ Process at CLIC

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ABSTRACT: Pair production of tau leptons in two photon collision $\gamma\gamma \rightarrow \tau^+\tau^-$ is studied at CLIC to test CP violating QED couplings of tau leptons. CP violating effects are investigated using tau pair spin correlations which are observed through the hadronic decay of each τ into $\pi\nu$. Competitive bounds with previous works on the electric dipole moment from CP odd terms have been obtained.

KEYWORDS: Tau physics, CP violation.

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1. Introduction

Although several recent results, including discovery of Higgs boson have been obtained from Large Hadron Collider (LHC), there has still been physics unexplored concerning CP violation. Information from Standard Model (SM) is not enough to have deeper understanding of the origin of CP violation. In leptonic interactions there is no CP violating couplings but it is possible that multi-loop contributions from quark sector [1, 2] indirectly induce CP violation which is too small to detect. Extensions of the Standard Model which cover neutrino mixing with different neutrino masses [3], more Higgs multiplets [4] could introduce CP violating couplings into the lepton sector. One of the motivations to go beyond the Standard Model comes from the baryon asymmetry in the universe. Supersymmetry [5], Left-Right Symmetric models [6], leptoquark models [7] have been proposed for additional sources of CP violation. There is no detectable electric dipole form factors of leptons in the Standard Model. If one considers coupling of leptons beyond the Standard Model, electric dipole form factors may cause detectable size of CP violation. Tau lepton is expected to have larger effects because of its large mass. In this work, we are interested in CP violating effects using the following parametrization for the effective electromagnetic coupling of tau lepton at the vertex $\gamma\tau\tau$ as an addition to electric charge γ^μ coupling [8]:

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m}\sigma^{\mu\nu}q_\nu + F_3(q^2)\frac{1}{2m}\sigma^{\mu\nu}q_\nu\gamma^5 \quad (1.1)$$

$$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) \quad (1.2)$$

where q and m are the momentum transfer to the photon and the mass of tau lepton. $F_1(q^2)$, $F_2(q^2)$ and $F_3(q^2)$ are electric charge, anomalous magnetic dipole and electric dipole form factors. In the limiting case of $q^2 \rightarrow 0$, the form factors are called moments which describe the static properties of the fermions.

$$F_1(0) = 1, \quad a_\tau = F_2(0), \quad d_\tau = \frac{e}{2m} F_3(0) \quad (1.3)$$

In the case of electron and muon, anomalous electromagnetic moments have been measured or constrained with high accuracy at low energy spin precession experiments. Because of the higher mass and short life time, experiments with tau lepton need colliders. This causes additional uncertainty in identification of tau lepton during production and decay processes when compared to low mass leptons. Accurate SM calculation with three loops gives tau lepton electric dipole moment (EDM) of the order of 10^{-35} e cm [1, 2]. Since this result is far from the present experimental capability, any observation at colliders about tau EDM shows the indication of new physics beyond the Standard Model. CP violating electric dipole form factor leads to contribution to the cross sections proportional to F_3^2 or higher order even power in many cases. CP even terms in cross sections can not be considered as a test of CP violation. A true CP violating contribution should appear linear or odd power in F_3 in the cross section which results from the interference with the Standard Model amplitude.

Upper limits on the electric dipole moment of the τ lepton with CP even terms in the cross sections have been obtained so far from the experiments at LEP [9, 10, 11]

$$|d_\tau| < 3.1 \times 10^{-16} \text{ e cm (L3)} \quad (1.4)$$

$$|d_\tau| < 3.7 \times 10^{-16} \text{ e cm (OPAL)} \quad (1.5)$$

$$|d_\tau| < 3.7 \times 10^{-16} \text{ e cm (DELPHI)}. \quad (1.6)$$

BELLE Collaboration used spin correlation observables to obtain limits on the electric dipole moment with CP odd terms in the cross section through the process $e^+e^+ \rightarrow \gamma \rightarrow \tau^+\tau^-$ [12]

$$-0.22 < \text{Re}(d_\tau) < 0.45 \text{ (} 10^{-16} \text{ e cm)} \quad (1.7)$$

$$-0.25 < \text{Im}(d_\tau) < 0.08 \text{ (} 10^{-16} \text{ e cm)}. \quad (1.8)$$

In this process, the intermediate photon has virtuality of $Q^2 = 100 \text{ GeV}^2$. Then, their bounds on real and imaginary parts of d_τ were given separately. There are several articles providing limits on electric dipole moment from previous LEP results or by using some indirect methods. Here we are going to mention only about the articles giving results based on terms linear in d_τ or CP odd terms in the cross sections [13].

In this paper, we are going to discuss the potential of CLIC to constrain EDM of tau lepton with CP odd terms in the cross section via tau pair production process in two photon collision $\gamma\gamma \rightarrow \tau^+\tau^-$. Direct production of tau pair $e^+e^- \rightarrow \tau^+\tau^-$ has small cross section due to s-channel Feynman amplitude with CLIC energies 500-3000 GeV in comparison to BELLE with center of mass energy around 10 GeV Fig.1-a. Another disadvantage of

direct production is connected to the photon virtuality $Q^2 = 10^5 - 10^7 \text{ GeV}^2$ which is not convenient for the definitions of electromagnetic moments with $Q^2 \rightarrow 0$. Pair production of charged leptons in two photon collision provide a unique opportunity to test Quantum Electrodynamics (QED). This process is followed by $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ via t-channel subprocess $\gamma\gamma \rightarrow \tau^+\tau^-$ based on equivalent photon approximation (later sections) Fig.1-b. Additional benefit from two photon collision is the proper photon virtuality $Q^2 \rightarrow 0$ at two vertices containing anomalous couplings. In this case, contrary to s-channel, t-channel contribution to the cross section increases with increasing energy due to sum over photon spectrum. As will be shown later, direct production cross sections are smaller than the case of two photon induced production of tau pair by a factor of $10^{-3} - 10^{-4}$ for CLIC energies.

The spin averaged cross section brings even powers of F_3 in the result that means CP is even. CP odd terms appear if spin dependent cross section is taken into account. Then we will need an observable which selects only CP odd terms from the cross section. In the next section, we give some details of the spin correlated cross section in the sub process $\gamma\gamma \rightarrow \tau^+\tau^-$ with the anomalous couplings of the τ lepton at both vertices. In section 3 we define azimuthal asymmetry as the spin correlated observable in terms of the final decay products of tau leptons. Section 4 deals with the equivalent photon approximation and the bounds on the electric dipole moment obtained for CLIC parameters. In the last section, we discuss the points concerning the background and tau lepton identification.

2. Spin Correlated Cross Section For Tau Pair production

For the subprocess $\gamma\gamma \rightarrow \tau^+\tau^-$, Feynman diagrams of t and u channels, as seen in Fig.1-b, are responsible where anomalous couplings are included by both vertices. Center of mass system of tau pairs is taken into account for calculation. Since spin dependent cross section is required, convenient coordinate system with z-axis along the produced τ^- lepton is chosen. In this frame, spin vectors of tau leptons in four dimensions can be given as

$$s_-^\mu = \left(\frac{p_{1z}}{m} s_{1z}, s_{1x}, s_{1y}, \frac{E}{m} s_{1z} \right) \quad (2.1)$$

$$s_+^\mu = \left(\frac{-p_{1z}}{m} s_{2z}, s_{2x}, s_{2y}, \frac{E}{m} s_{2z} \right) \quad (2.2)$$

where \vec{s}_1 and \vec{s}_2 are spin vectors of tau leptons in the rest frame of each tau. p_{1z} , and E are momentum and energy of τ^- , respectively. The squared amplitude are written explicitly in terms of reduced amplitudes and tau spins with cartesian components [14]

$$\begin{aligned} |M_1|^2 &= \frac{16\pi^2\alpha^2}{(\hat{t} - m^2)^2} [C_0 + C_{xx}s_{1x}s_{2x} + C_{yy}s_{1y}s_{2y} + C_{zz}s_{1z}s_{2z} \\ &\quad + C_{xy}^-(s_{1x}s_{2y} - s_{1y}s_{2x}) \\ &\quad + C_{yz}^-(s_{1y}s_{2z} - s_{1z}s_{2y}) + C_{xz}^+(s_{1x}s_{2z} + s_{1z}s_{2x})] \\ |M_2|^2 &= \frac{16\pi^2\alpha^2}{(\hat{u} - m^2)^2} [D_0 + D_{xx}s_{1x}s_{2x} + D_{yy}s_{1y}s_{2y} + D_{zz}s_{1z}s_{2z} \\ &\quad + D_{xy}^-(s_{1x}s_{2y} - s_{1y}s_{2x}) \end{aligned} \quad (2.3)$$

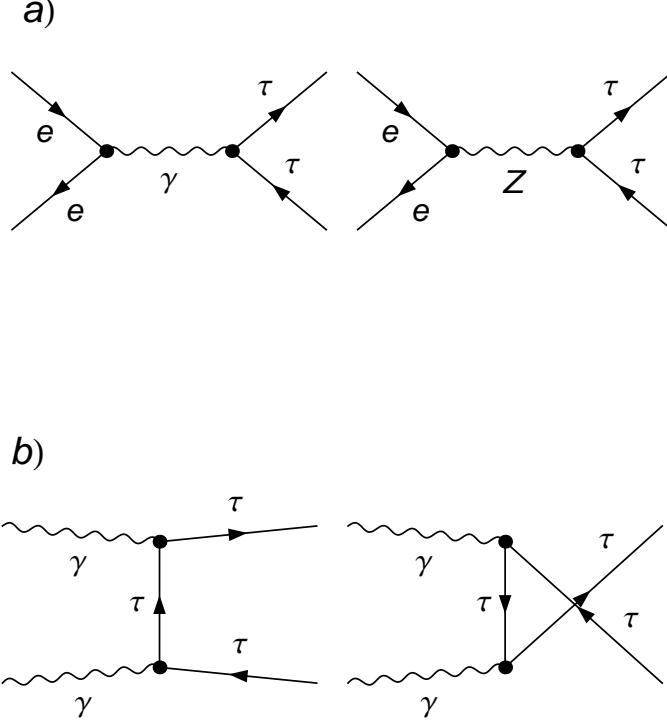


Figure 1: a) Direct production of tau pair at CLIC. b) Two photon induced tau pair production with equivalent photon approximation.

$$+D_{yz}^-(s_{1y}s_{2z} - s_{1z}s_{2y}) + D_{xz}^+(s_{1x}s_{2z} + s_{1z}s_{2x})] \quad (2.4)$$

$$|M_{12}| + |M_{21}| = \frac{16\pi^2\alpha^2}{(\hat{t} - m^2)(\hat{u} - m^2)} [G_0 + G_{xx}s_{1x}s_{2x} + G_{yy}s_{1y}s_{2y} + G_{zz}s_{1z}s_{2z} \\ + G_{xy}^-(s_{1x}s_{2y} - s_{1y}s_{2x}) \\ + G_{yz}^-(s_{1y}s_{2z} - s_{1z}s_{2y}) + G_{xz}^+(s_{1x}s_{2z} + s_{1z}s_{2x})] \quad (2.5)$$

where definitions of the quantites C_{ij}, D_{ij}, G_{ij} , $i, j = 0, x, y, z$ are given in the Appendix. In above expressions k_1, k_2, p_1 and p_2 are the momenta of the incoming photons and final τ leptons. Mandelstam variables are defined as $\hat{s} = (k_1 + k_2)^2$, $\hat{t} = (k_1 - p_1)^2$ and $\hat{u} = (k_1 - p_2)^2$. In the terms C_{ij}, D_{ij}, G_{ij} there are $F_3 F_1^3$, $F_3 F_2 F_1^2$, $F_3^3 F_1$ and terms with even powers of F_3 . Azimuthal asymmetry (next section) will select odd powers of F_3 terms in the numerator. However, the dominant term will be $F_3 F_1^3$ where $F_1 = 1$ is pointlike coupling, then we have retained F_3 and F_3^2 parts and higher orders F_3^3, F_3^4 have been neglected due to their smallness. There is also a term $F_2 F_1^3$ that does not contribute to azimuthal asymmetry. We should notice that \vec{s}_1 and \vec{s}_2 appear bilinearly which means two spins are correlated. Some of terms with anomalous coupling have spin correlations too.

The terms proportional to F_3 are CP odd and spin correlated. It is interesting that when spin correlation is removed CP odd terms vanish. Then we need convenient observables to determine size of CP violation within this process. When tau pair decays into charged particles and neutrinos, measuring spin correlation transforms to angular distributions of decay products. First we examine tau decay rate independently from pair production in the frame where tau lepton has momentum p_1 . For the decay mode to a charged hadron and a neutrino $\tau \rightarrow h + \nu$

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega_h} = \frac{1}{4\pi} \frac{m^2}{(E - \vec{p}_1 \cdot \hat{p}_h)^2} [1 - \vec{V}_h \cdot \vec{s}_1] \quad (2.6)$$

with

$$\vec{V}_h = \frac{1}{(E - \vec{p}_1 \cdot \hat{p}_h)} \left[\left(1 - \frac{\vec{p}_1 \cdot \hat{p}_h}{E + m}\right) \vec{p}_1 - m \hat{p}_h \right] \quad (2.7)$$

where \hat{p}_h shows unit momentum vector of hadron such as charged pion. From here on pion masses will be neglected. If the rest frame of tau is needed, \vec{p}_1 is taken to be zero and one gets familiar result

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega_h} = \frac{1}{4\pi} [1 + \alpha \hat{p}_h \cdot \vec{s}_1], \quad \alpha = \pm 1 \text{ for charged pions.} \quad (2.8)$$

In the subprocess $\gamma\gamma \rightarrow \tau^-\tau^+ \rightarrow (h^-\nu)(h^+\bar{\nu})$ tau decays occur through tau propagators. Around resonance, narrow width approximation is convenient to discuss correlation between decay products caused by tau spins. It is possible to combine tau pair production and decay parts to give following form [14]

$$\frac{d\hat{\sigma}}{d\cos\theta} = \frac{BR(\tau^- \rightarrow h^-\bar{\nu})BR(\tau^+ \rightarrow h^+\nu)}{16\pi^2} \int d\Omega_{h^-} d\Omega_{h^+} [A + \sum_{i,j} B_{ij} V_{h^-}^i V_{h^+}^j]. \quad (2.9)$$

The indices i, j indicate cartesian components x,y,z. The angle θ is between tau lepton and incoming photon in the center of mass system of tau pair. The contents of A , one of B_{ij} and relations between them are given below

$$A = \frac{\pi\alpha^2}{2\hat{s}} \left[\frac{C_0}{(\hat{t} - m^2)^2} + \frac{D_0}{(\hat{u} - m^2)^2} - \frac{G_0}{(\hat{t} - m^2)(\hat{u} - m^2)} \right] \quad (2.10)$$

$$B_{xy} = \frac{\pi\alpha^2}{2\hat{s}} \left[\frac{C_{xy}^-}{(\hat{t} - m^2)^2} + \frac{D_{xy}^-}{(\hat{u} - m^2)^2} - \frac{G_{xy}^-}{(\hat{t} - m^2)(\hat{u} - m^2)} \right] \quad (2.11)$$

$$B_{yx} = -B_{xy} \quad (2.12)$$

$$B_{zy} = -B_{yz} \quad (2.13)$$

$$B_{zx} = B_{xz}. \quad (2.14)$$

3. Azimuthal Asymmetry

In order to single out CP odd terms proportional to F_3 azimuthal asymmetry definition can be made using the angles of decay products. Because tau momenta is in the z-direction, azimuthal angles of decay products do not change even if the boost along tau momenta is applied. Then, integrations over azimuthal angles can be performed in a convenient reference system. Remaining integrations over polar angles will be done using full angular regions. Therefore, we replace $V_{h-}^i V_{h+}^j$ by $\hat{p}_{h-}^i \hat{p}_{h+}^j$ for simplicity. The terms proportional to F_3 in the cross section appear as $B_{xy}(\hat{p}_{h-}^x \hat{p}_{h+}^y - \hat{p}_{h-}^y \hat{p}_{h+}^x)$ with the following momentum directions

$$\hat{p}_{h-} = (\sin \theta_- \cos \phi_-, \sin \theta_- \sin \phi_-, \cos \theta_-) \quad (3.1)$$

$$\hat{p}_{h+} = (\sin \theta_+ \cos \phi_+, \sin \theta_+ \sin \phi_+, \cos \theta_+). \quad (3.2)$$

The integration over azimuthal angles ϕ_- and ϕ_+ can be organized according to signature of $\delta = \sin(\phi_- - \phi_+)$ based on the expression in the integrand below

$$(\hat{p}_{h-}^x \hat{p}_{h+}^y - \hat{p}_{h-}^y \hat{p}_{h+}^x) = \sin \theta_- \sin \theta_+ \sin(\phi_- - \phi_+). \quad (3.3)$$

It is required to use azimuthal asymmetry which compares number of decay products scattered normal to the left and to the right regions of the production plane of tau pair as defined by signature of δ

$$A_{az} = \frac{\int_{\delta>0} d\sigma - \int_{\delta<0} d\sigma}{\int_{\delta>0} d\sigma + \int_{\delta<0} d\sigma}. \quad (3.4)$$

This asymmetry keeps only CP violating terms with F_3 in the numerator. After performing integrals over angular parts of decay products in the denominator, we are left with expression without spin contributions such as C_0, D_0 and G_0 which include an additional terms with F_3^2 and higher order even powers. When compared with standard model part, we ignore F_3^2 and higher even powers in the denominator. While azimuthal asymmetry is taken into account, all angular integrations belong to decay products can be easily done in analytic way. Integrations over incoming photon spectrum and $\cos \theta$ belongs to tau pair production will be taken numerically.

4. Limits on CP Violation at CLIC

CLIC has a potential to produce almost real gamma beam from electron and positron beams with beam energy $E_b = 250 - 1500$ GeV. Therefore, two photon induced events at CLIC have large cross sections with final tau leptons. For additional integrations over each photon spectrum we consider equivalent photon approximation based on the work [15]

$$\begin{aligned} \frac{dN}{dE_\gamma} = f(x) = & \frac{\alpha}{\pi E_b} \left[\left(\frac{1-x+x^2/2}{x} \right) \log \frac{Q_{max}^2}{Q_{min}^2} - \frac{m_e^2 x}{Q_{min}^2} \left(1 - \frac{Q_{min}^2}{Q_{max}^2} \right) \right. \\ & \left. - \frac{1}{x} (1-x/2)^2 \log \frac{x^2 E_b^2 + Q_{max}^2}{x^2 E_b^2 + Q_{min}^2} \right] \end{aligned} \quad (4.1)$$

where m_e is the electron mass, $x = E_\gamma/E_b$ and Q_{max}^2 is the maximum virtuality. Kinematic definition of Q_{min}^2 can be given by

$$Q_{min}^2 = \frac{m_e^2 x^2}{1-x}. \quad (4.2)$$

Using the above function for both beams $f_1(x_1)$ and $f_2(x_2)$ with incoming photon energies E_1 and E_2 , the cross section becomes

$$d\sigma = \int dE_1 dE_2 f_1(x_1) f_2(x_2) d\hat{\sigma}(\hat{s}) \quad (4.3)$$

$$= \int \frac{dL^{\gamma\gamma}}{dW} d\hat{\sigma}(W) dW \quad (4.4)$$

$$\hat{s} = W^2. \quad (4.5)$$

Here W is the invariant mass of incoming photons and $dL^{\gamma\gamma}/dW$ is defined as the effective photon luminosity

$$\frac{dL^{\gamma\gamma}}{dW} = \int_{y_{min}}^{y_{max}} dy \frac{W}{2y} f_1\left(\frac{W^2}{4y}\right) f_2(y) \quad (4.6)$$

$$y_{min} = \max\left(\frac{W^2}{4E_2^{max}}, E_1^{min}\right), \quad y_{max} = E_1^{max}. \quad (4.7)$$

If scattered electrons of the beams are detected, maximum and minimum values of incoming photon energies can be determined experimentally. Otherwise final energy or momentum cuts of produced tau pair will be used to specify minimum photon energy. In order to get idea about the features of effective photon luminosity a graph of Eq.(4.6) has been drawn in Fig.2 for CLIC energies of $\sqrt{s}=500$ GeV, 3000 GeV [16, 17, 18] and for $Q_{max}^2 = 2$ GeV². As seen from this figure, lower values of incoming photon energies are much more effective on the cross section. In fact, %50 of cross section of the process can be reached by the energy $W_{max} = 0.03\sqrt{s}$. Thus, effective photon energy W for the cross section becomes less than 15-90 GeV depending on electron beam energy.

For small values of Q_{max}^2 and small collider energies, the last term in Eq.(4.1) is less effective and may be negligible but small x values, large collider energies and large Q_{max}^2 the last term becomes effective and reduces the cross sections considerably. There are several papers which use the code CompHep/CalcHep [19] or similar packages to compute cross sections. All these packages use Weizsaecker-Williams photon spectrum $f(x)$ without the last term in Eq.(4.1). In order to see Q_{max}^2 dependence with and without third term we give Standard Model $ee \rightarrow ee\tau\tau$ cross sections at CLIC in Table 1 for different Q_{max}^2 values. It is understood that full photon spectrum leads smaller cross sections and smoother behaviour against Q_{max}^2 . After imposing a transverse momentum cut on the final tau leptons the difference between the full and approximate photon spectrum gets smaller. In

order to make a comparison, Standard Model cross sections of direct production of tau lepton pair $ee \rightarrow \tau\tau$ are given below:

$$\sigma = 0.45 \text{ pb for } \sqrt{s} = 500 \text{ GeV} \quad (4.8)$$

$$\sigma = 0.01 \text{ pb for } \sqrt{s} = 3000 \text{ GeV} \quad (4.9)$$

where direct production cross sections are smaller by a factor of $10^{-3} - 10^{-4}$ than the values shown in Table 1.

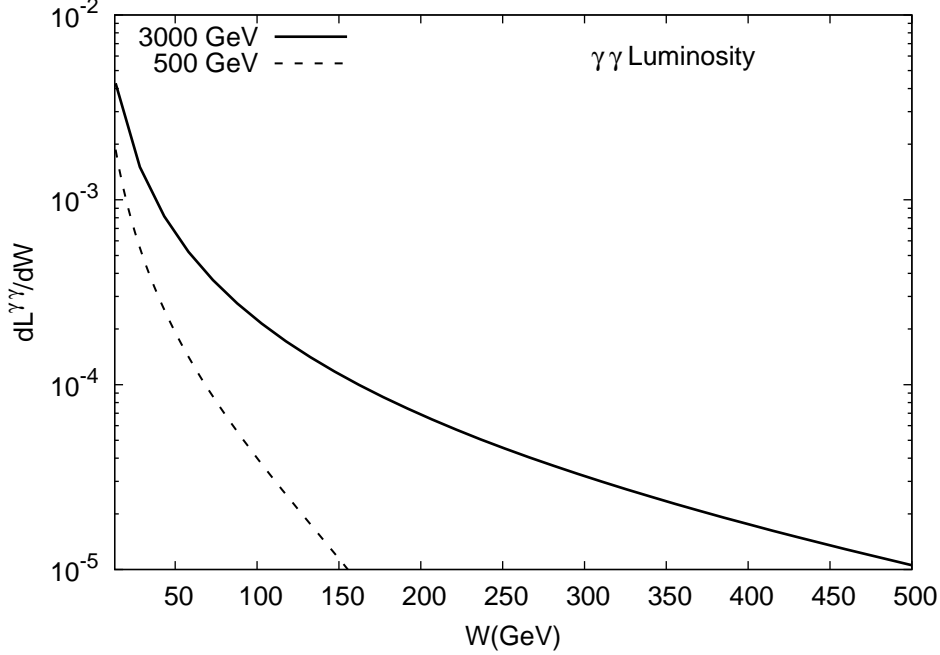


Figure 2: Effective $\gamma\gamma$ luminosity as a function of the invariant mass of the two photon system for electron beam energies $\sqrt{s} = 500, 3000 \text{ GeV}$ and $Q_{max}^2 = 2 \text{ GeV}^2$

After having information about equivalent photon approximation, azimuthal asymmetry as a function of $\cos\theta$ between tau leptons and incoming photons in the center of mass system of tau leptons has been obtained from Eq.(3.4) for tau decay products of $\pi^\mp \nu$. During computations energy and transverse momentum cuts 0.2 GeV on each final state charged pion were applied. Its properties are given by Fig.3 for $\sqrt{s} = 500 \text{ GeV}$ and $\sqrt{s} = 3000 \text{ GeV}$. According to Fig.3, highest asymmetry occurs when tau leptons are produced transverse to incoming photons direction. If integration over $\cos\theta$ is completed we get almost the same asymmetry $A_{az}/F_3 = 0.70$ for the energies of $\sqrt{s} = 500, 1500, 3000 \text{ GeV}$

Bounds on CP violating anomalous electric dipole moment d_τ can be found by χ^2 analysis using azimuthal asymmetry A_{az}

$$\chi^2 = \frac{(A_{az}(F_3) - A_{az}(F_3 = 0))^2}{\delta_{SM}^2} \quad (4.10)$$

$$\delta_{SM} = \sqrt{(\delta^{st})^2 + (\delta^{sys})^2} \quad (4.11)$$

Q_{max}^2 (GeV ²)	\sqrt{s} (GeV)	σ_{SM} (pb)	σ_{SM}^{PT} (pb)
2	500	332 (360)	56(58)
2	1500	523 (578)	99(103)
2	3000	667 (744)	131(139)
16	500	371 (428)	65(70)
16	1500	581 (680)	112(123)
16	3000	737 (869)	148(163)
64	500	392 (478)	70(78)
64	1500	613 (753)	120(136)
64	3000	776 (958)	158(181)

Table 1: Standard Model cross sections of $ee \rightarrow ee\tau^+\tau^-$ for CLIC energies and maximum virtuality Q_{max}^2 values. The numbers in parenthesis show cross sections obtained from the photon spectrum neglecting the last term in Eq.(4.1). In the last column $W_{min} = 2\sqrt{m_\tau^2 + P_T^2}$ have been considered with a cut $P_T = 5$ GeV of each tau lepton.

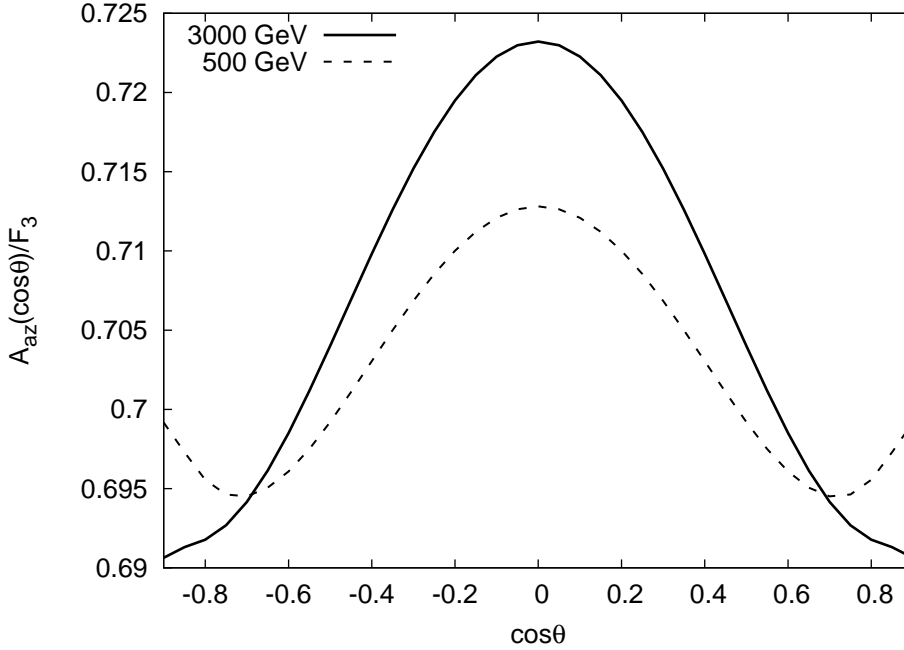


Figure 3: Azimuthal asymmetry $A_{az}(\cos\theta)/F_3$ as a function of polar angle $\cos\theta$ between tau leptons and incoming photon direction in the center of mass system of tau leptons.

$$\delta^{st} = \frac{1}{\sqrt{N_{SM}}} \quad (4.12)$$

$$N_{SM} = L_{int}\sigma_{SM}BR \quad (4.13)$$

where σ_{SM} , BR, N_{SM} and δ_{SM} are cross section, branching ratio, number of events and uncertainty without anomalous couplings. L_{int} is the integrated luminosity of CLIC. δ^{st} and δ^{sys} are statistical and systematic uncertainties. Table 2 shows the constraints on the CP odd anomalous electric dipole moment of the tau lepton which were computed

using different CLIC luminosities and systematic uncertainties. Maximum virtuality of Weizsacker-Williams photons $Q_{max}^2 = 2 \text{ GeV}^2$ has been taken.

$L_{int}(fb^{-1})$	$\sqrt{s} \text{ (GeV)}$	$ d_\tau (\text{e cm})$	$ d_\tau (\text{e cm}), \delta_1^{sys}$	$ d_\tau (\text{e cm}), \delta_2^{sys}$
50	500	3.34×10^{-17}	3.69×10^{-17}	1.59×10^{-16}
100	500	2.36×10^{-17}	2.82×10^{-17}	1.57×10^{-16}
230	500	1.56×10^{-17}	2.20×10^{-17}	1.56×10^{-16}
100	1500	1.86×10^{-17}	2.50×10^{-17}	1.56×10^{-16}
200	1500	1.32×10^{-17}	2.08×10^{-17}	1.56×10^{-16}
320	1500	1.04×10^{-17}	1.90×10^{-17}	1.56×10^{-16}
200	3000	1.16×10^{-17}	1.94×10^{-17}	1.56×10^{-16}
400	3000	8.23×10^{-18}	1.76×10^{-17}	1.55×10^{-16}
590	3000	6.77×10^{-18}	1.69×10^{-17}	1.55×10^{-16}

Table 2: Sensitivity of the process $ee \rightarrow ee\tau^+\tau^- \rightarrow ee\pi^+\pi^-\nu\bar{\nu}$ to CP odd tau anomalous electric dipole moment d_τ at 95% C.L. for CLIC energies $\sqrt{s} = 500, 1500, 3000 \text{ GeV}$ and integrated luminosities given above. Total systematic uncertainties used in χ^2 function in the last two columns have been taken as $\delta_1^{sys} = 0.001$ and $\delta_2^{sys} = 0.01$

The largest statistics of τ leptons have been collected by e^+e^- B-factories (BELLE, BABAR) due to high cross section and luminosity. One of their strategy is to provide precision studies for basic τ properties as complementary to lower mass leptons. As given in the introduction, the most precise bounds on real and imaginary parts of CP odd electric dipole form factor were determined separately by BELLE collaboration [12] in the process $e^+e^- \rightarrow \gamma \rightarrow \tau^+\tau^-$ with photon virtuality $Q^2 = 100 \text{ GeV}^2$. In our results average value of virtuality Q^2 is pretty smaller than 2 GeV^2 . This means photons coupled to tau leptons are much close to real photons. Therefore CLIC bounds of this work do not have one to one correspondence to BELLE bounds. For $\sqrt{s} = 3000 \text{ GeV}$ and systematic uncertainty less than $O(10^{-3})$, the limits obtained in Table 2 can be considered to be complementary or competitive to the ones of BELLE. For systematic uncertainty larger than $O(10^{-2})$, statistics is dominated by systematic uncertainty. However, a firm decision can only be made after a detailed investigation of experimental conditions at CLIC. A theoretical study to find sensitivity of CLIC to electric dipole moment of τ lepton has been done recently in the paper [20] which contains comparable limits but only for CP even terms.

5. Discussion

The main physics background for two photon collision case comes from the s-channel direct production of tau leptons. Comparison of total cross sections has been made in the previous section for both cases, two photon induced t-channel production and s-channel direct production. Better way to discriminate them is to examine their transverse momentum distributions which is described briefly in Appendix B and is drawn in Fig.4. In this figure, different p_T properties of two photon induced tau pair production cross section $e^+e^- \rightarrow e^+e^-\tau^-\tau^+$ and direct s-channel production $e^+e^- \rightarrow \tau^-\tau^+$ are given. Remarkably

dominant behaviour belongs to two photon collision for $p_T < 150$ GeV . The upper curve increases up to a peak region around the half of the tau mass as p_T approaches origin and then decreases down to zero when $p_T \rightarrow 0$. This is connected to t-channel propagator where its denominator gets smaller as p_T goes to origin. This is called collinear enhancement. When a factor of p_T is combined with the propagator as in Eq.(B.1) (Appendix), the curve gives a maximum region about $p_T \simeq m/2$. If lepton mass vanishes the collinear singularity occurs. In our case, tau mass takes the enhancement under control. In addition, minimum transverse momentum cut of a few GeV (5 or more) is used for efficient tau identification. This p_T cut already excludes the peak region and the calculated sensitivities are safe. Another reason for overall increase of upper curve is based on the sum over \sqrt{s} photon spectrum where largest contributions of the subprocess $\gamma\gamma \rightarrow \tau^+\tau^-$ take place at small \sqrt{s} values. In the case of direct production only one energy value $\sqrt{s} = 3000$ GeV contributes.

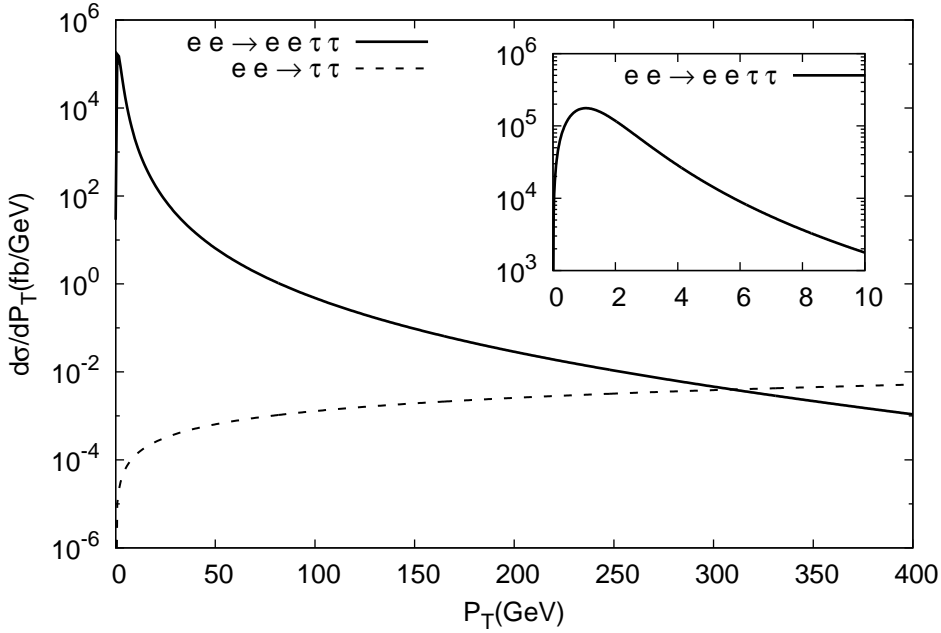


Figure 4: Transverse momentum distributions of tau pair. Two photon induced production of SM process with integration over photon spectrum (upper curve) and direct tau pair production (lower curve) at $\sqrt{s} = 3000$ GeV. The same curve due to two photon induced production is shown inside the small frame around tau mass region in order to see the typical behaviour that distribution goes to zero when $p_T \rightarrow 0$.

Another quantity is the invariant mass of tau pair which differs appreciably in two cases. In two photon induced process, invariant mass of tau pair should be equal to effective invariant mass of two photons which is less than 15-90 GeV. Invariant mass of tau pair in the s channel case is the same as incoming e^+e^- center of mass energy 500-3000 GeV with much smaller cross sections. As explained above, in small p_T region, background from direct production case can easily be eliminated. In addition, if scattered electrons with

small angles are tagged, two photon induced production cross section will have relatively clear background.

An important point is to detect final tau leptons with an uncertainty as small as possible. This depends on the decay products of tau lepton beside the experimental tools. In the subprocess $\gamma\gamma \rightarrow \tau^-\tau^+$, produced tau pair will not be back to back because of initial photon energies which are not the same with each other. In hadronic tau decay $\tau \rightarrow \pi\nu$, charged pions will be detected with its momentum direction while neutrino escapes detection. For tau lepton energies $m \ll E$, the decay products will be collimated with opening angle of the order of m/E . It is interesting that opening angle is completely determined by the mass and energy of the hadron. Both of them are measurable and angle is calculated from the decay kinematics [21]. When scattered electrons are tagged, initial photon energies can be determined by the difference of incoming and outgoing beam energies $E_\gamma = E_b - E'_b$. Then, the invariant mass of τ pair is obtained through photons momenta. Next, the τ decay products are boosted into the τ pair rest frame with boost parameter $(E_1^\gamma - E_2^\gamma)/(E_1^\gamma + E_2^\gamma)$ where the energy of each τ is defined in terms of the invariant mass of tau pair. Therefore, the tau momentum can be constrained to be in a cone defined by a fixed angle to the momentum of its hadronic decay product. Possible solutions for τ momenta correspond to intersections of two cones belongs to each τ . In the case of events containing single neutrino from each τ decay, almost the same methods have been used for lepton colliders [22, 23] with high efficiency without the need of knowledge about τ production vertex.

If deflected electrons and positrons are not required to be observed, the information about invariant mass of τ pair is lost due to missing momentum of the neutrino. In this case, tau reconstruction is still possible if production vertex of τ pair and charged prong trajectory are known sufficiently well. Let us consider the average decay length of tau lepton $L = \beta\gamma\tau_\tau$ in terms of relativistic factor $\gamma = 1/(\sqrt{1-\beta^2})$ and tau life time τ_τ . For the τ energy $E > 20$ GeV the decay length reaches mm size which leads to the measurable quantity using impact parameter method. Impact parameter is the shortest distance to the charged hadron momentum direction from the tau pair production vertex (or primary vertex). Impact parameter is crucial to determine secondary vertex (τ decay point) and can be written approximately $d = L\psi$ in terms of small opening angle ψ for sufficient τ energy. This method works well to determine neutrino momentum when single neutrino and single charged pion are produced [24].

In both cases above, full reconstruction of produced tau leptons is possible if single neutrino appears for each tau decay. To conclude, tau pair production in two photon collision at CLIC is sensitive to test CP odd EDM coupling of tau lepton especially for $\sqrt{s} = 3000$ GeV.

A. Definitions of Reduced Amplitudes

Definitions of reduced amplitudes in Eqs. 2.3-2.5 :

$$C_0 = 6m^4 - 2m^2(2\hat{s} + 5\hat{t} + 3\hat{u}) + 2\hat{t}\hat{u}$$

$$+F_3^2[\frac{1}{4m^2}((m^2 - \hat{t})(9m^4 + m^2(8\hat{s} - 22\hat{t} + 4\hat{u}) + \hat{t}(9\hat{t} - 4\hat{s})))] \quad (\text{A.1})$$

$$D_0 = 6m^4 - 2m^2(2\hat{s} + 3\hat{t} + 5\hat{u}) + 2\hat{t}\hat{u} \\ +F_3^2[\frac{1}{4m^2}((m^2 - \hat{u})(9m^4 + m^2(8\hat{s} - 22\hat{u} + 4\hat{t}) + \hat{u}(9\hat{u} - 4\hat{s})))] \quad (\text{A.2})$$

$$G_0 = 4(\hat{s}m^2 - 4m^4) \\ +F_3^2[\frac{1}{8m^2}(-44m^6 + m^4(32\hat{s} + 34(\hat{t} + \hat{u})) \\ -2m^2(16\hat{s}^2 + 8\hat{s}(\hat{t} + \hat{u}) - 9\hat{t}^2 + 20\hat{t}\hat{u} - 9\hat{u}^2) + 5\hat{s}^3 \\ +8\hat{s}^2(\hat{t} + \hat{u}) - 5\hat{s}(\hat{t} - \hat{u})^2 - 2(4\hat{t}^3 + \hat{t}^2\hat{u} + \hat{t}\hat{u}^2 + 4\hat{u}^3))] \quad (\text{A.3})$$

$$C_{xx} = 4m^2(2k_{1x}^2 + m^2 + \hat{t}) \\ +F_3^2[\frac{1}{4m^2}(m^2(14k_{1x}^2\hat{s} - 3\hat{s}^2 + 4\hat{s}\hat{t} + 3\hat{u}(\hat{u} - 2\hat{t})) \\ +\hat{t}(2k_{1x}^2\hat{s} - \hat{s}^2 + \hat{u}^2) - m^6 + m^4(12\hat{s} + 5\hat{t} - 2\hat{u}))] \quad (\text{A.4})$$

$$C_{yy} = 4m^2(m^2 + \hat{t}) + F_3^2[\frac{1}{4m^2}(m^2(-3\hat{s}^2 + 4\hat{s}\hat{t} + 3\hat{u}(\hat{u} - 2\hat{t})) \\ +\hat{t}(-\hat{s}^2 + \hat{u}^2) - m^6 + m^4(12\hat{s} + 5\hat{t} - 2\hat{u}))] \quad (\text{A.5})$$

$$C_{zz} = 4E^2(2k_{1z}^2 - 4k_{1z}p_{1z} + m^2 + 2p_{1z}^2 + \hat{t}) + 4p_{1z}^2(m^2 + \hat{t}) \\ +F_3^2[\frac{1}{4m^4}\{E^2(m^2(14k_{1z}^2\hat{s} - 4k_{1z}p_{1z}(5\hat{s} + 6\hat{t} - 2\hat{u}) \\ +p_{1z}^2(54\hat{s} - 56\hat{t} + 40\hat{u}) - 3\hat{s}^2 + 4\hat{s}\hat{t} - 6\hat{t}\hat{u} + 3\hat{u}^2) \\ +\hat{t}(2k_{1z}^2\hat{s} + 4k_{1z}p_{1z}(\hat{s} + 2\hat{u}) + 2p_{1z}^2(5\hat{s} + 4\hat{u}) \\ -\hat{s}^2 + \hat{u}^2) + m^4(-56k_{1z}p_{1z} - 56p_{1z}^2 + 12\hat{s} \\ +5\hat{t} - 2\hat{u}) - m^6) - p_{1z}^2(m^6 + m^4(-12\hat{s} - 5\hat{t} + 2\hat{u}) \\ +m^2(3\hat{s}^2 - 4\hat{s}\hat{t} + 6\hat{t}\hat{u} - 3\hat{u}^2) + \hat{t}(\hat{s}^2 - \hat{u}^2))\}] \quad (\text{A.6})$$

$$C_{xy}^- = F_3[-4E(k_{1z} - p_{1z})(3m^2 + \hat{t})] \quad (\text{A.7})$$

$$C_{yz}^- = F_3[-\frac{4E^2}{m}k_{1x}(3m^2 + \hat{t})] \quad (\text{A.8})$$

$$C_{xz}^+ = 8Ek_{1x}m(k_{1z} - p_{1z}) \\ +F_3^2[\frac{1}{2m^3}\{Ek_{1x}(m^2(7k_{1z}\hat{s} - p_{1z}(\hat{s} + 2\hat{t} - 6\hat{u})) \\ +\hat{t}(k_{1z}\hat{s} + p_{1z}(\hat{s} + 2\hat{u})) - 22m^4p_{1z}\}] \quad (\text{A.9})$$

$$D_{xx} = 4m^2(2k_{1x}^2 + m^2 + \hat{u}) \\ +F_3^2[\frac{1}{4m^2}(m^2(14k_{1x}^2\hat{s} - 3\hat{s}^2 + 4\hat{s}\hat{u} + 3\hat{t}(\hat{t} - 2\hat{u})) \\ +\hat{u}(2k_{1x}^2\hat{s} - \hat{s}^2 + \hat{t}^2) - m^6 + m^4(12\hat{s} + 5\hat{u} - 2\hat{t}))] \quad (\text{A.10})$$

$$D_{yy} = 4m^2(m^2 + \hat{u}) + F_3^2[\frac{1}{4m^2}(m^2(-3\hat{s}^2 + 4\hat{s}\hat{u} + 3\hat{t}(\hat{t} - 2\hat{u})) \\ +\hat{u}(-\hat{s}^2 + \hat{t}^2) - m^6 + m^4(12\hat{s} + 5\hat{u} - 2\hat{t}))] \quad (\text{A.11})$$

$$D_{zz} = 4E^2(2k_{1z}^2 + 4k_{1z}p_{1z} + m^2 + 2p_{1z}^2 + \hat{u}) + 4p_{1z}^2(m^2 + \hat{u}) \\ +F_3^2[\frac{1}{4m^4}\{E^2(m^2(14k_{1z}^2\hat{s} + 4k_{1z}p_{1z}(5\hat{s} + 6\hat{u} - 2\hat{t})$$

$$\begin{aligned}
& +p_{1z}^2(54\hat{s} - 56\hat{u} + 40\hat{t}) - 3\hat{s}^2 + 4\hat{s}\hat{u} - 6\hat{t}\hat{u} + 3\hat{t}^2) \\
& +\hat{u}(2k_{1z}^2\hat{s} - 4k_{1z}p_{1z}(\hat{s} + 2\hat{t}) + 2p_{1z}^2(5\hat{s} + 4\hat{t}) \\
& -\hat{s}^2 + \hat{t}^2) + m^4(56k_{1z}p_{1z} - 56p_{1z}^2 + 12\hat{s} \\
& +5\hat{u} - 2\hat{t}) - m^6) - p_{1z}^2(m^6 + m^4(-12\hat{s} - 5\hat{u} + 2\hat{t}) \\
& +m^2(3\hat{s}^2 - 4\hat{s}\hat{u} + 6\hat{t}\hat{u} - 3\hat{t}^2) + \hat{u}(\hat{s}^2 - \hat{t}^2))) \} \} \quad (A.12)
\end{aligned}$$

$$D_{xy}^- = F_3[4E(k_{1z} + p_{1z})(3m^2 + \hat{u})] \quad (A.13)$$

$$D_{yz}^- = F_3[-\frac{4E^2}{m}k_{1x}(3m^2 + \hat{u})] \quad (A.14)$$

$$\begin{aligned}
D_{xz}^+ &= 8Ek_{1x}m(k_{1z} + p_{1z}) \\
& +F_3^2[\frac{1}{2m^3}\{Ek_{1x}(m^2(7k_{1z}\hat{s} + p_{1z}(\hat{s} + 2\hat{u} - 6\hat{t})) \\
& +\hat{u}(k_{1z}\hat{s} - p_{1z}(\hat{s} + 2\hat{t})) + 22m^4p_{1z}\}) \} \} \quad (A.15)
\end{aligned}$$

$$\begin{aligned}
G_{xx} &= 2(k_{1x}^2(8m^2 - 4\hat{s}) + 2m^4 + 4m^2(\hat{t} + \hat{u}) + \hat{s}^2 - \hat{t}^2 - \hat{u}^2) \\
& +F_3^2[\frac{1}{8m^2}(4k_{1x}^2(36m^4 - 10m^2(\hat{s} + 4m^2) + 3\hat{s}^2 + 7\hat{s}\hat{t} + 7\hat{s}\hat{u} \\
& +(\hat{t} + \hat{u})^2) - 20m^6 + 10m^4(8\hat{s} + 3\hat{t} + 3\hat{u}) + 2m^2(4\hat{s}^2 - 8\hat{s}\hat{t} - 8\hat{s}\hat{u} \\
& -5\hat{t}^2 - 20\hat{t}\hat{u} - 5\hat{u}^2) - 3\hat{s}^3 - 4\hat{s}^2\hat{t} - 4\hat{s}^2\hat{u} \\
& +3\hat{s}\hat{t}^2 + 10\hat{s}\hat{t}\hat{u} + 3\hat{s}\hat{u}^2 + 4\hat{t}^3 + 6\hat{t}^2\hat{u} + 6\hat{t}\hat{u}^2 + 4\hat{u}^3)] \quad (A.16)
\end{aligned}$$

$$\begin{aligned}
G_{yy} &= 2(2m^4 + 4m^2(\hat{t} + \hat{u}) + \hat{s}^2 - \hat{t}^2 - \hat{u}^2) \\
& +F_3^2[\frac{1}{8m^2}(-20m^6 + 10m^4(8\hat{s} + 3\hat{t} + 3\hat{u}) + 2m^2(4\hat{s}^2 - 8\hat{s}\hat{t} - \hat{s}\hat{u} \\
& -5(\hat{t}^2 + 4\hat{t}\hat{u} + \hat{u}^2) - 3\hat{s}^3 - 4\hat{s}^2(\hat{t} - 4\hat{s}^2\hat{u} + \hat{s}(3\hat{t}^2 + 10\hat{t}\hat{u} + 3\hat{u}^2) \\
& +4\hat{t}^3 + 6\hat{t}^2\hat{u} + 6\hat{t}\hat{u}^2 + 4\hat{u}^3)] \quad (A.17)
\end{aligned}$$

$$\begin{aligned}
G_{zz} &= \frac{2}{m^2}\{E^2(2k_{1z}^2(8m^2 - 4\hat{s}) + 8k_{1z}p_{1z}(\hat{t} - \hat{u}) + 2m^4 \\
& +4m^2(6p_{1z}^2 + \hat{t} + \hat{u}) - 12p_{1z}^2\hat{s} - 8p_{1z}^2\hat{t} - 8p_{1z}^2\hat{u} + \hat{s}^2 - \hat{t}^2 - \hat{u}^2) \\
& +p_{1z}^2(2m^4 + 4m^2(\hat{t} + \hat{u}) + \hat{s}^2 - \hat{t}^2 - \hat{u}^2)\} \\
& +F_3^2[\frac{1}{8m^4}\{E^2(4k_{1z}^2(36m^4 - 10m^2(3\hat{s} + 2\hat{t} + 2\hat{u}) + 3\hat{s}^2 + 7\hat{s}\hat{t} \\
& +7\hat{s}\hat{u} + (\hat{t} + \hat{u})^2) - 8k_{1z}p_{1z}(\hat{t} - \hat{u})(-8m^2 + \hat{s} + 4\hat{t} + 4\hat{u}) - 20m^6 \\
& +m^4(80\hat{s} + 30\hat{t} + 30\hat{u} - 304p_{1z}^2) - 2m^2(p_{1z}^2(76\hat{s} + 8\hat{t} + 8\hat{u}) \\
& -4\hat{s}^2 + 8\hat{s}\hat{t} + 8\hat{s}\hat{u} + 5\hat{t}^2 + 20\hat{t}\hat{u} + 5\hat{u}^2) \\
& +36p_{1z}^2\hat{s}^2 + 76p_{1z}^2\hat{s}\hat{t} + 76p_{1z}^2\hat{s}\hat{u} + 28p_{1z}^2\hat{t}^2 + 24p_{1z}^2\hat{t}\hat{u} + 28p_{1z}^2\hat{u}^2 - 3\hat{s}^3 \\
& -4\hat{s}^2\hat{t} - 4\hat{s}^2\hat{u} + 3\hat{s}\hat{t}^2 + 10\hat{s}\hat{t}\hat{u} + 3\hat{s}\hat{u}^2 + 4\hat{t}^3 \\
& +6\hat{t}^2\hat{u} + 6\hat{t}\hat{u}^2 + 4\hat{u}^3) + p_{1z}^2(-20m^6 + 10m^4(8\hat{s} + 3\hat{t} + 3\hat{u}) \\
& +2m^2(4\hat{s}^2 - 8\hat{s}\hat{t} - 8\hat{s}\hat{u} - 5\hat{t}^2 - 20\hat{t}\hat{u} - 5\hat{u}^2) - 3\hat{s}^3 - 4\hat{s}^2\hat{t} - 4\hat{s}^2\hat{u} \\
& +\hat{s}(3\hat{t}^2 + 10\hat{t}\hat{u} + 3\hat{u}^2) + 4\hat{t}^3 + 6\hat{t}^2\hat{u} + 6\hat{t}\hat{u}^2 + 4\hat{u}^3)\} \} \quad (A.18)
\end{aligned}$$

$$G_{xy}^- = F_3[4E(k_{1z}(\hat{t} - \hat{u}) + 4m^2p_{1z} + p_{1z}(5\hat{s} + 2\hat{t} + 2\hat{u}))] \quad (A.19)$$

$$G_{yz}^- = F_3[\frac{4E^2}{m}k_{1x}(\hat{t} - \hat{u})] \quad (A.20)$$

$$\begin{aligned}
G_{xz}^+ = & \frac{8E}{m}(k_{1x}(2k_{1z}m^2 - k_{1z}\hat{s} + p_{1z}\hat{t} - p_{1z}\hat{u}) \\
& + F_3^2[\frac{1}{2m^3}\{Ek_{1x}(k_{1z}(36m^4 - 10m^2(3\hat{s} + 2\hat{t} + 2\hat{u}) + 3\hat{s}^2 + 7\hat{s}(\hat{t} + \hat{u}) \\
& + (\hat{t} + \hat{u})^2) + 3p_{1z}\hat{s}(\hat{t} - \hat{u}))\}
\end{aligned} \tag{A.21}$$

where k_1 , k_2 , p_1 and p_2 are the momenta of the incoming photons and final τ leptons. Mandelstam variables are defined as $\hat{s} = (k_1 + k_2)^2$, $\hat{t} = (k_1 - p_1)^2$ and $\hat{u} = (k_1 - p_2)^2$. E and m are energy and mass of tau leptons.

B. Transverse Momentum Distribution

Transverse momentum p_T distribution of final tau leptons in the center of mass system of tau pair is calculated using the following expressions :

$$\frac{d\hat{\sigma}}{dp_T} = \frac{p_T |M|^2 \delta(y_\tau - y_0)}{8\hat{s}\pi(2\sqrt{\hat{s}}E_T \sinh y_0)} dy_\tau \tag{B.1}$$

$$y_0 = \text{arccosh}\left(\frac{\sqrt{\hat{s}}}{2E_T}\right) \tag{B.2}$$

$$E_T = \sqrt{p_T^2 + m^2} \tag{B.3}$$

where y_τ is the rapidity of tau lepton:

$$y_\tau = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}. \tag{B.4}$$

Mandelstam variables in the Spin averaged square of the Feynman amplitude $|M|^2$ can also be written in terms of p_T and y_τ

$$\hat{t} = m^2 - \sqrt{\hat{s}}E_T e^{-y_\tau} \tag{B.5}$$

$$\hat{u} = m^2 - \sqrt{\hat{s}}E_T e^{y_\tau} \tag{B.6}$$

For the case of two photon collision two more integrations over photon spectrum are needed.

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